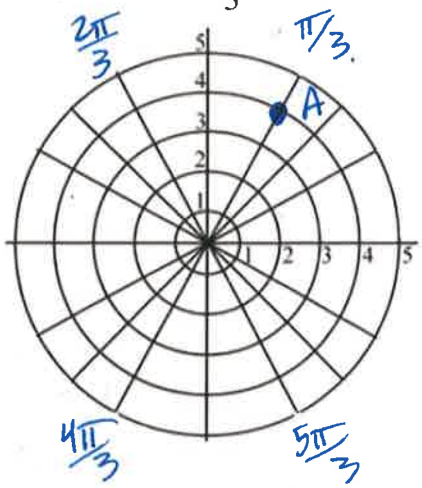
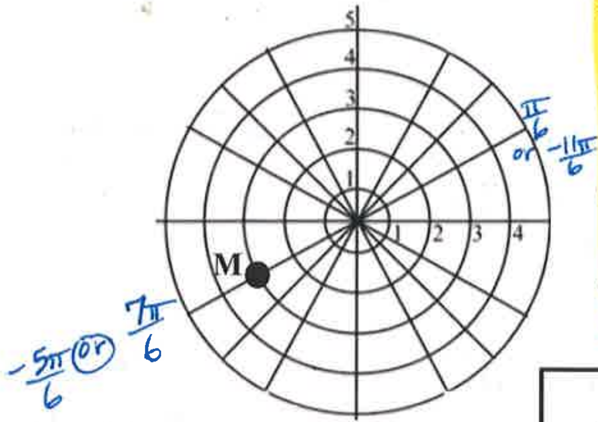
Ch.8 Review#1—NO CALCULATOR!!   Name: Key   Per:

1. Graph the point $(-4, \frac{4\pi}{3})$ and label it A.

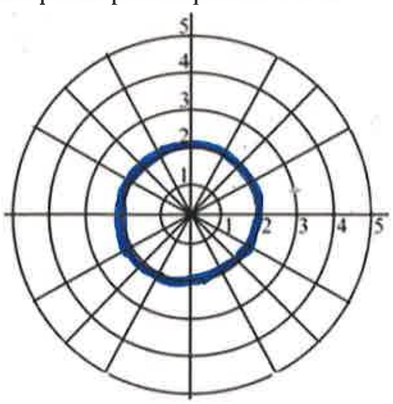


2. Fill in each blank to name four possible coordinates for point M. $-2\pi \le \theta \le 2\pi$

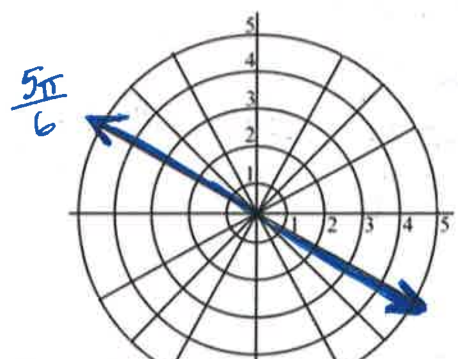
a. $(3, \frac{7\pi}{6})$
b. $(3, \frac{-5\pi}{6})$ ok to switch
c. $(-3, \frac{\pi}{6})$
d. $(-3, \frac{-11\pi}{6})$ ok to switch



3. Graph the polar equation r = 2



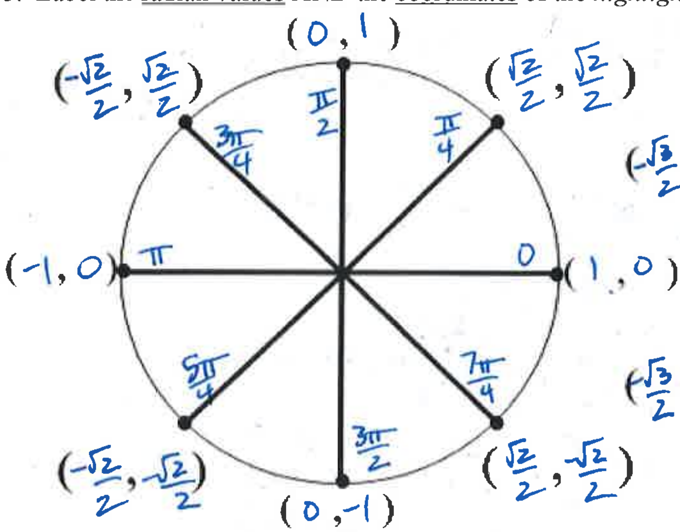
4. Graph the polar equation $\theta = \frac{5\pi}{6}$

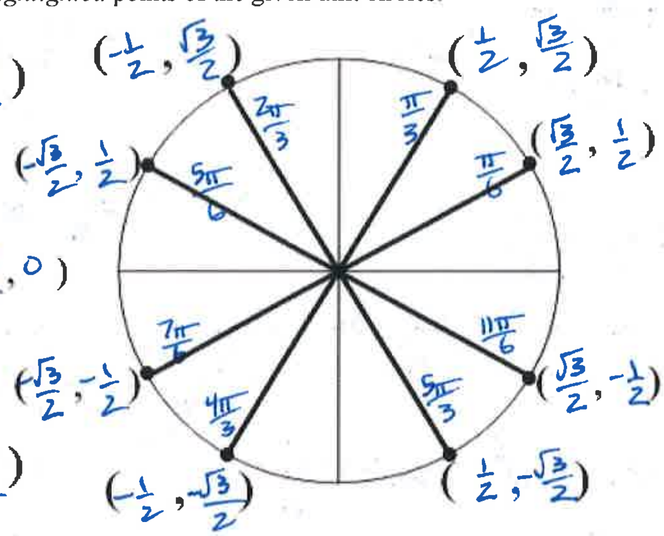


CHECK ANSWERS #2, 6-13

$\cancel{-1}$ $\cancel{0}$ $\cancel{1}$ $\cancel{-\frac{\sqrt{3}}{2}}$ $\cancel{\frac{\sqrt{2}}{2}}$ $\cancel{-\frac{11\pi}{6}}$ $\cancel{-\frac{5\pi}{6}}$ $\cancel{\pi}$ $\cancel{\frac{\pi}{6}}$ $\cancel{\frac{\pi}{6}}$ $\cancel{\frac{7\pi}{6}}$ $\cancel{\frac{3\pi}{4}}$ $\cancel{\frac{5\pi}{4}}$ $\cancel{\frac{5\pi}{3}}$

5. Label the <u>radian values</u> AND the <u>coordinates</u> of the *highlighted* points of the given unit circles.





Evaluate using *exact* answers from the unit circle.

6. $\sin\frac{3\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

7. $\tan\frac{7\pi}{4} = \boxed{-1}$

8. $\cos\frac{7\pi}{6} = \boxed{\frac{-\sqrt{3}}{2}}$

9. $\sin\frac{\pi}{2} = \boxed{1}$

For #10-11, solve for **<u>principal values</u>**. Then solve the general expressions in **#12,13 by finding two values** each for $0 \le \theta < 2\pi$.

10. $\text{Arctan}\left(\frac{-\cancel{4}\sqrt{3}}{\cancel{4}}\right) = \boxed{\frac{5\pi}{3}}$ IV
Arctan $(-\sqrt{3})$

11. $\text{Arcsin}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$ I

12. $\text{arccos}\left(\frac{-\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}, \frac{5\pi}{4}}$
general values

13. $\text{arctan}(0) = \boxed{0, \pi}$

Reminders: $r = \sqrt{x^2 + y^2}$ **or** $r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$ $x = r\cos\theta$ $y = r\sin\theta$

14. Find the ***polar*** coordinates of the point with rectangular coordinates (-5, 5). Be sure that $0 \le \theta < 2\pi$ for your final solution. Show work. *Express r as an exact value and θ in radians.*

Quad II

$r = \sqrt{(-5)^2 + 5^2}$

$r = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = \frac{5}{-5}$

$\tan\theta = -1$ $\theta = \frac{3\pi}{4}$

$\boxed{\left(5\sqrt{2}, \frac{3\pi}{4}\right)}$ r θ

15. Find the ***rectangular*** coordinates of the point with polar coordinates $\left(-2, \frac{4\pi}{3}\right)$. Show work and use exact values.

$x = -2\cos\frac{4\pi}{3}$

$x = -2\left(-\frac{1}{2}\right)$

$x = 1$

$y = -2\sin\frac{4\pi}{3}$

$y = -2\left(-\frac{\sqrt{3}}{2}\right)$

$y = \sqrt{3}$

$\boxed{(1, \sqrt{3})}$ x y

Simplify #16-18. Show all steps.

16. $(8 - i) - 3(-1 + 5i)$

$= 8 - i + 3 - 15i$

$= \boxed{11 - 16i}$

17. $(2 + 5i)^2 =$

$= (2+5i)(2+5i)$

$= 4 + 20i + 25i^2$

$= 4 + 20i - 25 = \boxed{-21 + 20i}$

18. $\frac{6+2i}{-2+i} \cdot \frac{(-2-i)}{(-2-i)} = \frac{-12-10i-2i^2}{4-i^2}$

$= \frac{-12-10i+2}{4+1}$

$= \frac{-10-10i}{5} = \boxed{-2-2i}$

19. Express $4 + 4i$ in polar form. Show work.
Hint: find r and θ.

(4,4) Quad I

$r = \sqrt{(-4)^2 + 4^2}$

$r = \sqrt{32}$

$r = 4\sqrt{2}$

$\tan\theta = \frac{4}{-4}$

$\tan\theta = -1$

$\theta = \frac{3\pi}{4}$

$\boxed{= 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}$

20. Express $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ in rectangular form. Show work.
Hint: simplify as is.

$= 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= \boxed{-\sqrt{3} + i}$

CHECK ANSWERS

~~$r\cos\theta$~~ ~~$x^2 + y^2$~~
~~$r\sin\theta$~~ ~~$x^2 + y^2$~~
~~$-21 + 20i$~~ ~~$\frac{y}{x}$~~
~~$-2 - 2i$~~
~~$-\sqrt{3} + i$~~
~~$\frac{3}{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$~~
~~$12\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$~~
~~$11 - 16i$~~
~~$(1, \sqrt{3})$~~
~~$\left(5\sqrt{2}, \frac{3\pi}{4}\right)$~~
~~$4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$~~
~~$\frac{3}{2}$~~ ~~12~~ ~~4096~~
~~$\frac{3\pi}{4}$~~ ~~$\frac{11\pi}{6}$~~
~~$x = 8$~~
~~$r = 12\csc\theta$~~
~~$r = 2\cos\theta$~~
~~$x^2 + y^2 - 2y = 0$~~

21. Identify the modulus and the argument (show work), then find the product. Express answer in polar form.
$4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \cdot 3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

modulus: $4 \cdot 3 = \boxed{12}$

argument: $\frac{2\pi}{3} + \frac{7\pi}{6} = \frac{4\pi}{6} + \frac{7\pi}{6} = \boxed{\frac{11\pi}{6}}$

$= \boxed{12\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)}$

$= 12\left(\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right)$

$= 6\sqrt{3} - 6i$ rectangular form

22. Identify the modulus and the argument (show work), then find the quotient. Express answer in polar form.
$6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) \div 4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

modulus: $\frac{6}{4} = \boxed{\frac{3}{2}}$

argument: $\frac{3\pi}{2} - \frac{3\pi}{4} = \frac{6\pi}{4} - \frac{3\pi}{4} = \boxed{\frac{3\pi}{4}}$

$= \boxed{\frac{3}{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}$

$= \frac{3}{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$= -\frac{3\sqrt{2}}{4} + \frac{3\sqrt{2}}{4}i$ rectangular form

23. Use De Moivre's Theorem to find $\left(2 + 2\sqrt{3}i\right)^6$. Express your result in rectangular form. Show work.

$r = \sqrt{2^2 + (2\sqrt{3})^2}$

$r = \sqrt{4 + 12}$

$r = \sqrt{16} = 4$

$\tan\theta = \frac{2\sqrt{3}}{2}$ Quad I

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$= 4^6\left[\cos 6\left(\frac{\pi}{3}\right) + i\sin 6\left(\frac{\pi}{3}\right)\right]$

$= 4096(\cos 2\pi + i\sin 2\pi)$

$= 4096(1 + 0i) = \boxed{4096}$

Write the given equation in polar form. (*HINT: use substitution to solve.*)

24. $y = 12$

$r\sin\theta = 12$

$r = \frac{12}{\sin\theta}$ $\boxed{r = 12\csc\theta}$

25. $x^2 + y^2 - 2x = 0$

$r^2 - 2r\cos\theta = 0$

$r^2 = 2r\cos\theta$ $\boxed{r = 2\cos\theta}$

Write the given equation in rectangular form. (*HINT: use substitution to solve.*)

26. $r^2 - 2r\sin\theta = 0$

$\boxed{x^2 + y^2 - 2y = 0}$

27. $r = \frac{8}{\cos\theta}$

$r\cos\theta = 8$

$\boxed{x = 8}$